Superluminal propagation of light in gravitational field and non-causal signals.

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Abstract

It has been found in several papers that, because of quantum corrections, light front can propagate with superluminal velocity in gravitational fields and even in flat space-time across two conducting plates. We show that, if this is the case, closed time-like trajectories would be possible and, in particular, in certain reference frames photons could return to their source of origin before they were produced there, in contrast to the opposite claim made in the literature.

1 Introduction

A study of photon propagation in gravitational field has revealed a surprising phenomenon that quantum corrections modify the characteristics of photon equation of motion in such a way that in some cases they may lay outside the light cone. This effect was first found in ref. [1] for several different geometries (gravitational wave, Schwarzschild, and Robertson-Walker). In subsequent papers it has been shown that photons may propagate "faster than light" also in the Reissner-Nordström [2] and the Kerr [3] backgrounds. Similar effect was found for propagation of massless neutrinos in gravitational field [4]. The only essential difference is that photons can propagate with $v > c$ even in vacuum where Riemann tensor is non-vanishing, $R_{\alpha\beta\mu\nu} \neq 0$, while neutrinos may acquire superluminal velocity only in

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space-time with $R_{\mu\nu} \neq 0$. Superluminal propagation has been also found for flat space-time with boundaries, e.g. for photon propagation between conducting plates [5, 6, 7] (see also [8]). In what follows we will discuss both possibilities and will show that in these cases one could find a coordinate frame where photons would return to their source before they were created there.

The photon effective action in vacuum in one loop approximation and in the lowest order in the gravitational field strength is described by the well known vacuum polarization diagram. Because of gauge invariance of electrodynamics and general covariance of gravity the result for the action is unique and can be written immediately:

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha C}{m_e^2} R_{\alpha\beta\mu\nu} F_{\mu\nu} F^{\alpha\beta} \right) \tag{1}$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, $\alpha = 1/137$ is the fine structure constant, m_e is the mass of electron and the coefficient C, as calculated in ref. [1], is $C = -1/360\pi$. For our purpose is essential just that $C \neq 0$, even its sign is not important. The action may also contain other terms proportional to the Ricci tensor $R_{\mu\nu}$ or to the curvature scalar R but they both vanish in vacuum and will be neglected in what follows.

It is evident that this action leads to the equation of motion with modified highest derivative terms and generally speaking the coefficient at ∂^2/∂_t^2 would not be equal to the one at ∂^2/∂_j^2 (with the evident factor g_{ij}/g_{tt}). In other words the characteristics of the photon wave equation with the corrections (1) generically would not coincide with the normal light cone. It means that the velocity of the propagation of the *front* of the signal would be modified. One should not worry if this happened to be inside the light cone but as we have already mentioned this is not the case. In particular, as was shown in ref. [1], in the Schwarzschild background photons going in a non-radial direction could propagate either faster than c or slower depending on their polarization. This may imply serious problems for the theory.

The velocity of light front propagation is known [9] to be determined by the asymptotics

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of the refraction index $n(\omega)$ for $\omega \to \infty$. Since the action (1) is believed to be valid only for sufficiently low photon frequency, $\omega \ll m_e$, the problem of superluminal propagation might be resolved by possible unaccounted for terms which vanish in the low frequency limit. It was argued in ref. [1] that if the refraction index satisfy dispersion relation with a positive definite $Im\, n(\omega)$, then $n(0) > n(\infty)$ and the problem persists. However it was shown [10] that positiveness of $Im\, n$ in gravitational fields is not necessarily true and one may hope that high frequency contributions would give rise to $n(\infty) > 1$ and correspondingly $v < c$.

Another twist to the problem was given by observation [11] that the expression for refraction index calculated from the effective action (1) is valid in the first order of gravitational interaction for *any* value of photon frequency. Indeed refraction index is determined by the forward scattering amplitude and the latter, in turn, is exactly given by the second term because forward scattering amplitude of a photon in the lowest order in external field can be only a function of photon 4-momenta squared, $k_1^2 = k_2^2 = 0$ and of the transferred momentum, $q^2 = (k_1 - k_2)^2$, which also vanishes for forward scattering. This simple argument invalidates the possibility of resolving the problem by higher order terms in ω/m_e mentioned in references [10, 12].

Thus in the lowest order in external classical gravitational field, in the first order of electromagnetic coupling α, and neglecting quantum gravity corrections the result (1) determines photon refraction index for all frequencies. In this approximation photons in a certain polarization state would propagate outside the normal light cone. In this connection the following two questions arise. First, if such superluminal propagation would create any problem with causality and, second, if higher order corrections in electromagnetic and gravitational interactions could return photons "back to normality".

The first question has been addressed in the literature (in practically all quoted here papers) and the almost unanimous conclusion is that, though there exist reference frames in which superluminal photons would reach detector, by the clock of an observer in this frame, earlier than they were emitted but it is impossible to send photons back to their source prior

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to their emission in the proper time of the source. In other words, non-causal signals would not be possible and, in particular, closed space-like trajectories for superluminal photons do not exist. This conclusion was based on the absence of Poincare invariance of the theory due to presence of a fixed gravitating center. We will show here that this conclusion is incorrect and explicitly construct an example of a non-causal closed space-like trajectory for a signal propagating outside the light cone. In view of that the problem seems to be considerably more grave than it was apprehended earlier.

Before presenting our example let us discuss what mechanisms or what changes in physics may in principle cancel the effect of superluminal propagation or prevent from traveling backward in time if the effect persists. An evident possibility is a contribution from higher order corrections in electromagnetic and gravitational interactions. Still it is not easy to achieve. Higher order electromagnetic corrections do not help. They could only give an extra power of α and no extra terms in the action proportional to derivatives of $F_{\mu\nu}$. It follows from dimensional consideration and from renormalizability of quantum electrodynamics in classical gravitational background. The structure of correction to the effective action remains the same, $\delta S \sim (RFF)/m_e^2$. Power counting shows that the diagrams with several external graviton legs (higher order corrections in external field) also do not give rise to derivative of electromagnetic field. The only dangerous diagrams are those with virtual gravitons, and only with at least two virtual gravitons. Such diagrams could give the terms of the following form:

$$\delta_2 S = C_2(\partial F \partial F R)/m_{Pl}^4 \tag{2}$$

Here m_{Pl}^4 is the Planck mass and in the expression in the brackets a proper contraction of indices is made.

Since the coefficient C_2 cannot be large (in fact it should be much smaller than unity), this contribution is not dangerous too for photon frequency below m_{Pl}. The loops with three virtual gravitons can give terms similar to (2) but now with the diverging coefficient

$C_3 \sim \Lambda^2/m_{Pl}^4$. Strictly speaking one cannot say anything about the magnitude of such terms but in superstring based theories of consistent renormalizable quantum gravity one would expect that $\Lambda^2 \sim m_{Pl}^2$ and in these theory (theories?) the contribution of higher order corrections from virtual gravitons would be negligible. Still it is worthwhile to understand more rigorously if the effect of superluminal propagation exists in a well defined quantum gravity.

Another possibility mentioned in the literature [1] is a modification of the usual causal light cone by an effective cone which corresponds to the fastest possible signal propagation. This would require quite serious changes in the usual physics and moreover it is not clear how it could be realized. The theory even with the modified effective action (1) remains Lorenz invariant as well as general covariant. It permits to express time running in a new reference frame through time and coordinates in the original frame. Introduction of a new causal cone with a non-constant maximum speed would probably result in a theory quite different from General Relativity. So we have either to admit that general relativity is broken at velocities very close to speed of light or to live in the world with non-causal signals.

2   Non-causal signals in Special Relativity

Let us remind how one can obtain acausal signal propagation in Special Relativity with "normal" tachyons moving with a constant speed u, exceeding speed of light c (which we take to be equal to 1). We assume that a tachyon is emitted at the point x_1 at the moment t_1 and is registered at the point x_2 at the moment t_2 of some inertial frame. In this reference frame the evident relation holds:

$$t_2 - t_1 = (x_2 - x_1)/u > 0 \tag{3}$$

Now let us make Lorenz transformation to another inertial frame moving with respect to the first one with velocity V:

$$x' = \gamma(x - Vt), \quad t' = \gamma(t - Vx) \tag{4}$$

where $\gamma = 1/\sqrt{1 - V^2}$.

In the new frame the time interval between emission and registration of the tachyon is given by:

$$t'_2 - t'_1 = \gamma(t_2 - t_1)(1 - Vu) \tag{5}$$

For $V = 1/u$ the time interval $(t'_2 - t'_1)$ can be zero, which corresponds to tachyon propagating in the second frame with infinite velocity and for a larger V the interval $(t'_2 - t'_1)$ could be even negative, i.e. the tachyon in this frame propagates backward in time. The absolute value of the tachyon velocity is always bigger than c, approaching $\pm\infty$ when V tends to $1/u$ from above or from below. Let us assume now that at the moment when tachyon reaches the detector at the point x_2 (or x'_2 in the second frame) this detector emits another tachyon back to the source. Let us assume for simplicity that the picture is symmetric so that the second source/detector emits a tachyon with the same velocity u with respect to itself and that both the first and the second emitters of tachyons move in the primed reference frame in opposite directions with equal velocities $V = 1/u$. Thus both tachyons would have infinite velocities in this frame and the signal would instantly return to the place of origin. It is evident that in the case of $V > 1/u$ both tachyons would travel into the past and the signal would return back to the first emitters "in less than no time".

Bearing in mind the examples presented below we will consider a slightly modified and more "realistic" construction of gedanken experiment with tachyons, which permits to avoid collision of tachyon emitters. Let us assume that there are two identical tachyon emitters A and B moving in the opposite directions along parallel straight lines (along x) separated by some distance Δy (see fig. 1).

A tachyon is emitted by the source A at the time moment t_0 and in the chosen reference

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frame it moves backward in time. At the moment t_1 $(t_1 < t_0)$ the tachyon reflects in perpendicular direction (now it moves along y) and at the moment t_2 $(t_1 < t_2 < t_0)$ it collides with the emitter B. Space-time picture of its motion (in terms of t and x with fixed y) is presented in fig. 2. The collision of the tachyon with B triggers the emission by the latter of another tachyon. This new tachyon moves again backward in time along the line of motion of B but in the opposite direction (all along x). At the moment t_3 it reflects perpendicular to x and moves to A and hits the latter at the moment t_4. It is evident that the system can be chosen so that $t_4 < t_0$. Space-time picture of the motion of the second tachyon is presented in fig. 3.

The possibility of sending signal, moving faster than light, back into the past to the source of its origin, which we have just demonstrated, is well known in the standard Special Relativity. It is normally assumed that tachyons move with a constant speed $u > 1$ independently of space points. In the case of propagation across two conduction plates it is not so, the velocity of photon can be bigger then c between the plates and equal to c outside [5, 6]. Still it is evident that the conclusion of acausal propagation would also survive in this case. Let us consider one dimensional motion along x and assume that the velocity of light is $v_l = 1$ for $|x| > d$ and $v_l = u > 1$ for $|x| < d$. For a realization of the gedanken experiment which we will discuss, a small hole should be made in the plates so that the photon could penetrate into the inner space. The size of the hole should be large in comparison with the wave length and simultaneously small not to destroy the effect of superluminal propagation. We will not go into these subtleties and consider this model as a toy model for illustration of a possible travel into the past. The arguments go essentially along the same lines as for "normal" tachyons. Let us assume as above that photon is emitted at the point x_1 at the moment t_1 and registered at x_2 at the moment t_2 in some inertial frame. On the way the photon passes the region between the plates where it moves faster than light with velocity

$u > 1$. In this reference frame the following evident relation holds:

$$t_2 - t_1 = x_2 - x_1 - 2d + 2d/u \tag{6}$$

We can again make the Lorenz transformation (4). The time of arrival of the signal in the primed coordinate frame to the point x_2' is given by the expression

$$t_2' - t_1' = \gamma(t_2 - t_1) \left(1 - V - \frac{2d}{t_2 - t_1} \frac{u - 1}{uV} \right) \tag{7}$$

Again the difference $(t_2' - t_1')$ may be negative for V sufficiently close to unity. It is interesting that the sign of the ratio $(t_2' - t_1')/(t_2 - t_1)$ depends upon the distance between the points 1 and 2.

The gedanken experiment with the return of the signal back to the emitter prior to the emission can be constructed in the same way as above with the standard tachyons which moves with a constant superluminal velocity. The space-time picture for such an "experiment" is presented in fig. 4. The world line of the signal in the primed system looks as following: the photon is "produced" out of nothing somewhere between his real birth place and the detector (the moment t'' in the figure) and propagates both ways to the place of birth and to the detector. When it reaches the moment t_0 another photon (original) is emitted from the source and "annihilates" with the first one at the moment t'.

Returning to arranging the time machine for photons in this conditions we can do essentially the same things as in the previously considered case with the only difference that now we have to supply both source/detectors A and B with their own plates with holes so that one set of plates moves together with A while the other moves with B .

3   Acausal signals in gravitational fields

In the case of superluminal photons traveling in a gravitational field the excess of the velocity over c is not given by a step function but continuously drops as $1/r^4$ with the increasing distance from the gravitating center. Evidently this does not create any serious difficulties.

What is more important is the curvature of space-time and the dependence of distance and time intervals upon the space-time metric and also the bending of the tachyon trajectory and the delay of signal due to interaction with gravitational field.

We will consider motion in spherically symmetric Schwarzschild background created by a localized matter not necessarily forming a black hole. The metric can be e.g. written in the form:

$$ds^2 = a^2(r)dt^2 - b^2(r)(dx^2 + dy^2 + dz^2) \tag{8}$$

where $r^2 = x^2 + y^2 + z^2$ and $a^2 = (1 - r_g/4r)^2/(1 + r_g/4r)^2$ and $b^2 = (1 + r_g/4r)^4$. However the explicit form of the metric functions a and b is not essential. A coordinate transformation to another reference frame moving at infinity with respect to the original one with velocity V has been considered in ref. [13]. The choice of coordinates made in this book is not convenient for our purposes so we will take another coordinate system which essentially coincides with that of ref. [13] at asymptotically large distances from the gravitating center [3].

As one of the spatial coordinate lines we will choose the trajectory of the superluminal photon in this metric and the coordinate running along this trajectory we denote as l. The other two, denoted $x_\perp$, are assumed to be orthogonal (in three dimensional sense). We will consider only motion along this trajectory so that assume that $x_\perp = 0$. The metric along this trajectory can be written in the form:

$$ds^2 = A^2(l)dt^2 - B^2(l)dl^2 \tag{9}$$

Now let us go to a different coordinate frame which moves with respect to the original one with velocity V along l at large distances from the center. The corresponding coordinate transformation can be chosen as

$$t' = \gamma \left[t - Vl - Vf(l) \right] \tag{10}$$

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[3]The inconvenience of the coordinate choice made in ref. [13] is related to the fact that for large V the coordinate frame has a physical singularity due to peculiarity of its motion near the source of gravity.

$$l' = \gamma \left[l + f(l) - Vt \right] \tag{11}$$

where the function $f(l)$ is chosen in such a way so that the crossed terms $dt'dl'$ do not appear in the metric. One can easily check that this can be achieved if

$$f(l) = \int^{l} dl \left(\frac{B}{A} - 1 \right) \tag{12}$$

In this moving frame the metric takes the form:

$$ds^2 = A^2 \left(dt'^2 - dl'^2 \right) \tag{13}$$

where A should be substituted as a function of l' and t'.

The motion of the tachyonic photon in the original frame satisfies the condition

$$u = \frac{B dl}{A dt} = 1 + \delta u \tag{14}$$

The quantum correction to the speed of light δu is assumed to be positive. As was shown in ref. [1] it is indeed the case for certain photon polarization. In the case that the polarization turns out to correspond to subluminal propagation we can always put (in our gedanken experiment) a depolarizor to change the polarization to the superluminal one. Radially moving photons in Schwarzschild background has normal velocity, $u = c$, so that we have to choose a trajectory with a nonzero impact parameter. However in dilaton gravity, as shown in ref. [14], a superluminal photon velocity is possible even for radial trajectories. In this case the arguments proving an existence of time machine would be slightly simpler.

The correction to the velocity of light is extremely small, roughly it is

$$\delta u = \frac{\alpha C_v r_g \rho}{r^4 m_e^2} \tag{15}$$

where r is the distance to the gravitating source and ρ is the impact parameter. The coefficient C_v is numerically small, about $\alpha/30\pi \approx 10^{-4}$, but the main suppression comes form the enormously small factor $1/(rm_e)^2$.

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Correspondingly the time interval between emission and absorption of the superluminal photon is equal to

$$t_2 - t_1 = \int_{l_1}^{l_2} dl\,(1 - \delta u)\,(B/A) \tag{16}$$

As follows from expression (11) this time interval in the primed system is

$$t_2' - t_1' = \gamma \left[(1 - V) \int_{l_1}^{l_2} dl(B/A) - \int_{l_1}^{l_2} dl\delta u(B/A) \right] \tag{17}$$

Clearly for sufficiently small $(1 - V)$ this difference can be negative and the signal can propagate into the past. Its behavior as a function of the distance $(l_2 - l_1)$ is similar to that found above for the case of propagation between conduction plates (see eq. (6).

To arrange traveling backward in time we will do now the same trick as was described above for the case of flat space-time. We assume that in the primed system the photon detector moves symmetrically with respect to the photon source i.e. it moves together with the attached to it another gravitating body along the parallel path with the same velocity but in the opposite direction. The arguments proving the possibility of traveling backward in time in this frame exactly repeat the previous ones for the propagation across the plates.

It is worth noting that the gravitational field of the body moving together with the detector influences the motion of the tachyon and its source and vice versa. However for the motion with ultrarelativistic velocities this gravitational field is concentrated in a very thin plane moving together with the body and perpendicular to the direction of the motion. Thus the influence of this field on the magnitude of the tachyon velocity would be extremely short and hence negligible. The direction of the tachyon motion might be changed quite significantly but this change could be immediately corrected by a mirror or by an emission of a new tachyon with the same properties from the place of the collision with the field.

4 Conclusion

Thus we have shown that quantum corrections to the photon propagation in a curved space-time or across conducting plates, which lead to superluminal velocity, would indeed permit

travel backward in time, as one would naively expect. This statement is in contradiction with the previously published papers [1, 12] where it was claimed that though it would be possible in a certain coordinate frame to arrive to detector earlier than the photon had been emitted, still a return to the original emitter prior to the birth of the photon was impossible. This statement was based on the assumed absence of the Poincare invariance in the system under consideration. However, as one can see, the Poincare invariance still exists but in a slightly more complicated form, namely one should consider the photon source or detector together with the attached to each of them gravitating center as a single entity. After this observation it becomes practically evident that traveling backward in time due to quantum corrections is indeed possible.

Thus we face the following dilemma, either time machine is possible in principle or something is wrong in the conclusion of superluminal propagation due to quantum corrections (vacuum polarization). It was argued [15, 16] that one can still has a consistent physics even if time travel is possible. The other option that some unaccounted for effect may kill superluminal propagation and permit to return to normality, looks more conservative and for many people more natural. However at the moment it is not clear how this cure can be achieved. The analysis of higher order corrections seems to support the present conclusion of superluminal propagation.

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Figure Captions:

Fig. 1. Motion of tachyons in $(x - y)$-plane. A and B are tachyonic sources moving in in the opposite directions as indicated. Tachyon emitted by A at the moment t_0 moves backward in time and at the moment $t_1 < t_0$ it is reflected to B. The moment of collision of this tachyon with B triggers emission by B of another tachyon which moves along x again backward in time. At the moment t_3 it is reflected back to A and hits it at the moment t_4 before it was emitted by A, $t_4 < t_0$.

Fig. 2. Space-time trajectory of the first tachyon described in fig. 1.

Fig. 3. Space-time trajectory of the second tachyon described in fig. 1.

Fig. 4. Space-time trajectory of the tachyon moving between conducting plates. The plates are assumed to be displaced along y to avoid collision.
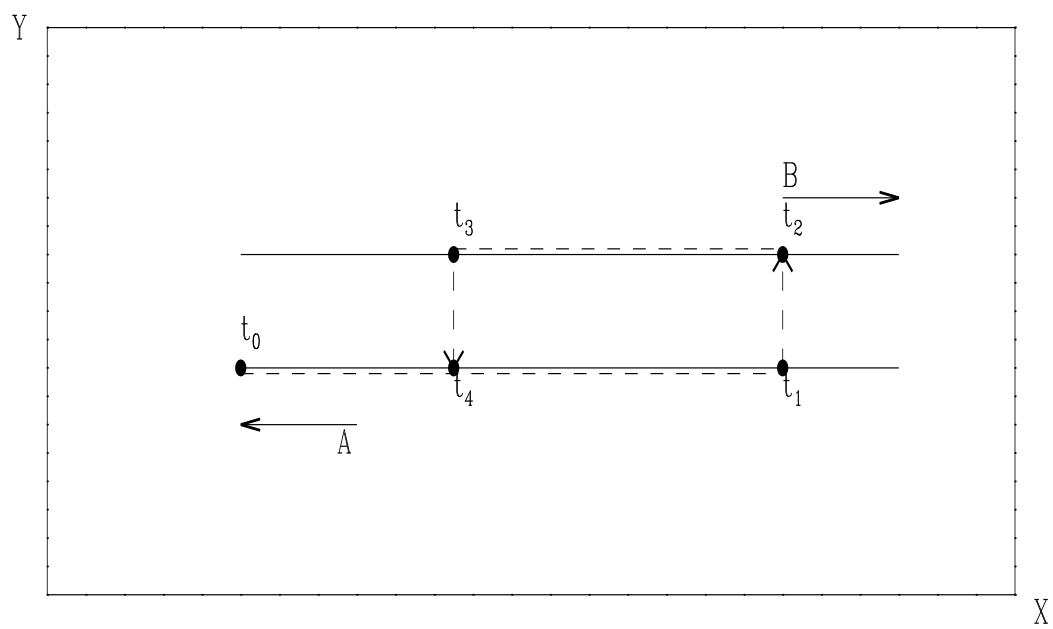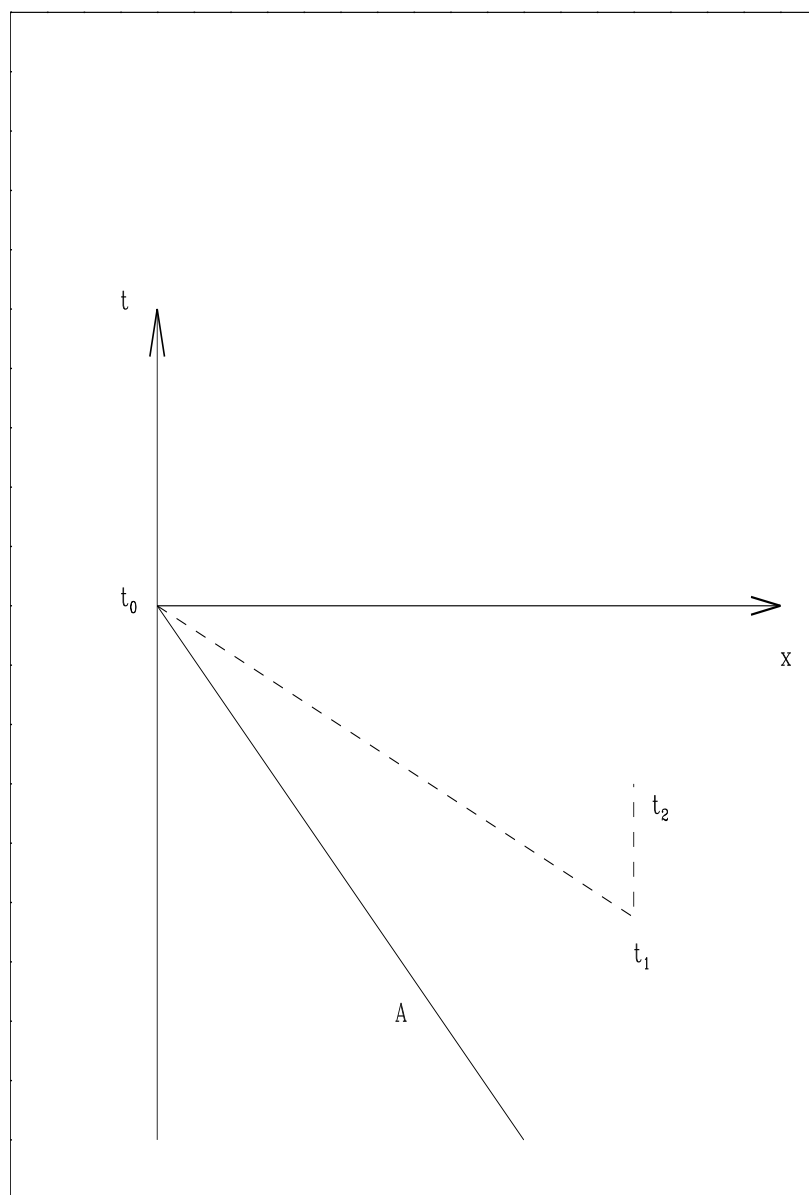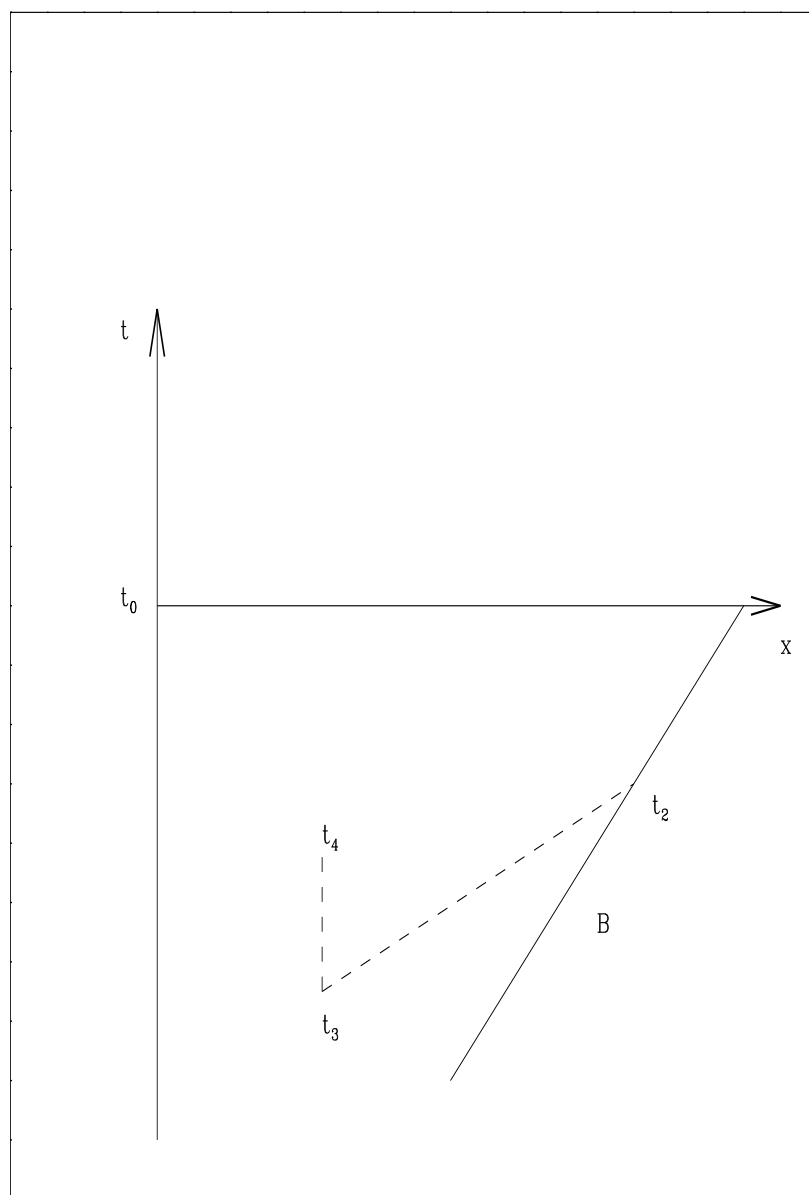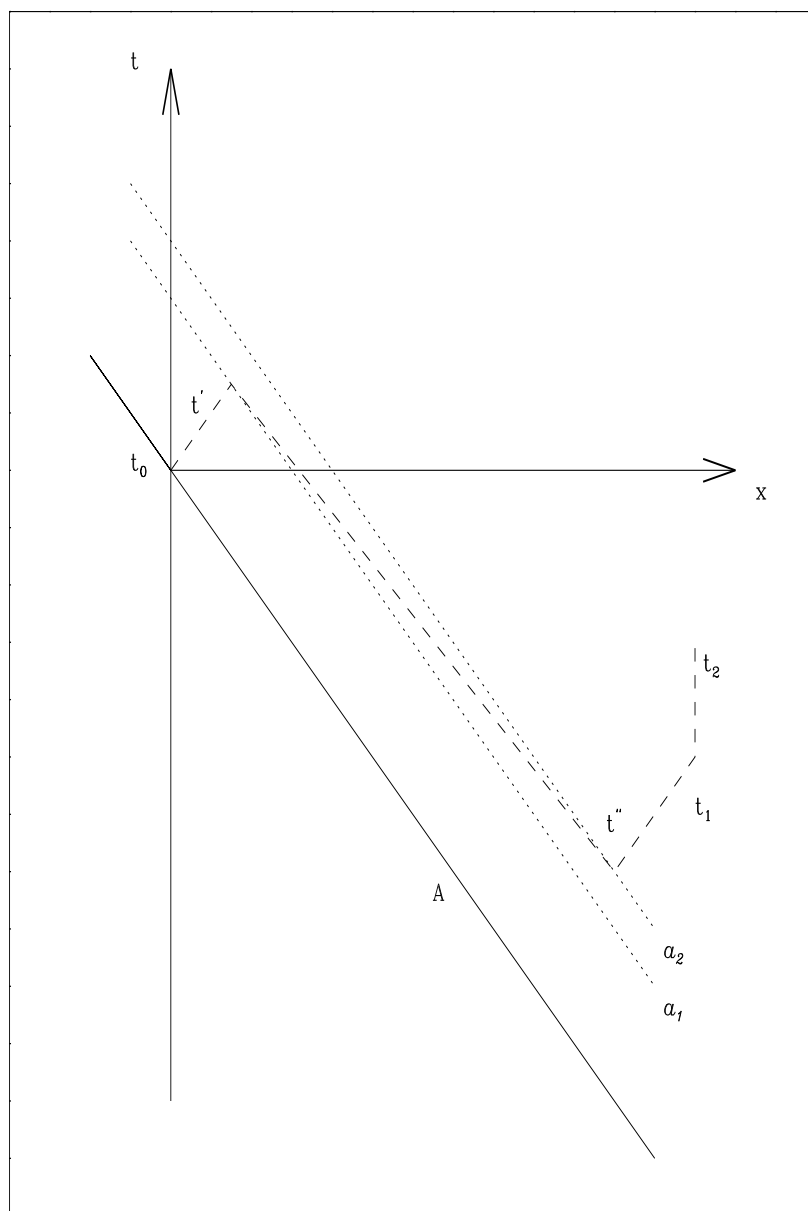
Figure 1.

Figure 2.

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